Particle models in connection with growing interfaces

Thanh Tam PHUNG , MAPMO, Orléans

Francois JAMES, MAPMO, Orléans

Pascal BRAULT, GREMI, Orléans

The Kuramoto-Sivashinsky (KS) equation is often used for describing interfaces with pseudo-periodical structures (patterning) [1, 2]:

$$\partial_t h(x,t) = -\nabla^2 h(x,t) - \nabla^4 h(x,t) + \frac{1}{2}(\nabla h(x,t))^2 \tag{1}$$

for which the typical length scale of the patterns k is $2\pi\sqrt{2}$. Because physics is also concerned by discrete systems, a particle version of this equation is particularly useful. In this work, we propose to compare the KS equation with two models presented in [3]. These models describe the dynamics in term of positions x_i, momenta p_i and acceleration $\dot{p}_i$ where $\dot{p}_i$ is the derivative with respect to time t. Within these models, there occur particle collisions and particle creations. In a collision two particles of mass 1 give one particle of mass 1. Creation occurs when the distance between two particles is larger than $\alpha > 0$, which plays a role similar to the above length scale k. A particle of mass 1 is created. Apart from the collision/creation process, the particles are subjected to one of the following dynamics. In a first attempt, we consider the one-dimensional particle model without acceleration, i.e

$$\dot{x}_i = p_i, \ \dot{p}_i = 0. \tag{2}$$

In this case this model was claimed to describe the so-called Burgers equation

$$\partial_t u(x,t) + u(x,t)\partial_x u(x,t) = 0 \tag{3}$$

However, it does not exhibit patterning. The second model includes a repulsive force $\dot{p}_i$ between the particles

$$\dot{x}_i = p_i, \ \dot{p}_i = 2x_i - x_{i+1} - x_{i-1}, \tag{4}$$

which leads to the equation

$$\partial_t u(x,t) + u(x,t)\partial_x u(x,t) = \dot{p}_i \frac{x - x_{i-1}}{x_i - x_{i-1}} + \dot{p}_{i-1} \frac{x_i - x}{x_i - x_{i-1}}. \tag{5}$$

We observe a more chaotic behavior. However, it remains not easy to make a clear connection between this model and KS equation. The goal is to find the partial differential equation that describes this particle model. Even the case of pure sticky particles without creation [4, 5], it is not obvious.

Références

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Thanh Tam PHUNG , MAPMO UMR 6628 Université d'Orléans BP 6759, Route de Chartres, 45067 Orléans cedex 2, France
`thanhtam.phung@math.cnrs.fr`
Francois JAMES, MAPMO UMR 6628 Université d'Orléans BP 6759, Route de Chartres, 45067 Orléans cedex 2, France
`francois.james@math.cnrs.fr`
Pascal BRAULT, GREMI, UMR6606 CNRS-Université d'Orléans BP 6744,45067 Orléans Cedex 2, France
`Pascal.Brault@univ-orleans;fr`