The relativistic version of Burgers equation

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First of all, we consider scalar, hyperbolic balance laws posed on a curved spacetime endowed with a volume form, and, after imposing a natural Lorentz invariance property (enjoyed by relativistic compressible fluids) we identify a unique (up to normalization) balance law. The latter can be viewed as a *relativistic version of Burgers equation,* and provides us with a drastically simplified model for the dynamics of relativistic compressible fluids. We then discuss here some fundamental properties of our model: hyperbolicity, genuine nonlinearity, stationary solutions. Second, we introduce a finite volume scheme for the approximation of solutions to this relativistic Burgers model. Our scheme is fully consistent with the divergence form of the equation and applies to weak solutions containing shock waves. Numerical experiments, presented with various choices of volume forms, demonstrate the convergence of the proposed finite volume scheme, and its relevance for computing long-time asymptotics of (possibly discontinuous) solutions in a curved background.

Références

[1] P. Amorim, M. Ben-Artzi, and P.G. LeFloch, *Hyperbolic conservation laws on manifolds: total variation estimates and the finite volume method*, Meth. Appl. Anal. 12 (2005), 291-324.

[2] M. Ben-Artzi, and P.G. LeFloch, *The well-posedness theory for geometry compatible hyperbolic conservation laws on manifolds*, Ann. Inst. H. Poincare : Nonlin. Anal. 24 (2007), 989-1008.

[3] P.G. LeFloch and B. Okutmustur, *Hyperbolic conservation laws on spacetimes with limited regularity*, C.R. Math. Acad. Sc. Paris 346 (2008), 539-543.

[4] P.G. LeFloch and B. Okutmustur, *Hyperbolic conservation laws on spacetimes. A finite volume schemes based on differential forms*, Far East J. Math. Sci. 31 (2008), 49-83.

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