

Numerical null controllability of the 1D heat equation: primal algorithms

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This work is concerned with the numerical computation of distributed null controls (with small support) for the 1D heat equation, with Dirichlet boundary conditions. The goal is to compute a control that drives (an approximation of) the solution from a prescribed initial state at $t = 0$ to zero at $t = T$. We deal with primal methods. More precisely, we minimize over the class of admissible null controls a functional that involves weighted integrals of the state and the control, with weights that blow up near T . In view of the results of Fursikov and Imanuvilov [4], the associated optimality conditions can be viewed as the weak formulation of a differential problem that is fourth order in space and second order in time. In a first method in this work, we introduce a finite element approximation of the optimality system that is C^1 in space and we perform some numerical experiments. The null controllability requirement (at $t = T$) is satisfied only approximately since, in this formulation, it is imposed indirectly, through appropriate Neumann conditions. In order to overcome this difficulty (and also in order to avoid the use of C^1 finite elements), we present a mixed variational formulation of the problem, we introduce a (non-conformal) C^0 finite element approximation and we provide new numerical results. With this second method, the initial and final conditions are satisfied exactly. The structure of the problem leads to ill-conditioning as the mesh size goes to zero. This motivates a third method, which relies on the augmented Lagrangian strategy (unfortunately, the standard techniques need again C^1 in space finite elements). We present several experiments and we discuss the robustness of this direct approach with respect to the size of the control support and the regularity of the diffusion coefficient.

Références

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