Integrators for Highly Oscillatory Hamiltonian Systems Using Homogenization

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Highly oscillatory Hamiltonian systems arise in applications such as molecular dynamics or celestial mechanics. One wishes to exploit the separation of time scales and create integrators whose time step is larger than the fastest frequencies. These integrators should capture well the slow motions and invariants of the systems without fully resolving the fast motions of the system.
This talk will present work building on [1]. We consider the Hamilton–Jacobi equations associated with the Newton equations of motion and apply a two-scale ansatz to the PDE. After expanding and deriving an approximate generating function satisfying the PDE, one has a symplectic algorithm for integrating the Hamiltonian equations.
Numerical tests of the resulting algorithm are presented, with comparison to other integrators for highly oscillatory systems. In the earliest schemes presented, resonances occurred when the integrator's time step was a constant multiple of one of the fast frequencies, and this destroyed the accuracy. The resonance behavior of our algorithm and related algorithms will be presented.

Références

[1] C. Le Bris et F. Legoll, *Integrators for Highly Oscillatory Hamiltionian Systems: An Homogenization Approach*, Discrete Contin. Dyn. Syst. Ser. B, 2010

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