Higher-Order Pyramidal Finite Elements for Electromagnetics

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Higher-order finite elements have exhibited a very good efficiency for hexahedral elements [1], but the automatic generation of unstructured hexahedral meshes is still challenging. A solution consists of generating hexahedral dominant meshes including a small number of tetrahedra, triangular prisms and pyramids. However, constructing pyramidal finite elements is not an easy task. In [3], we obtained the optimal finite element space for non-affine pyramids, and our approach is compared with other existing methods (e.g. [4],[2]). In this paper, only nodal basis functions are considered, and the method has mainly been studied for continuous finite elements and only applied for acoustics wave equation.

In the present talk, we will focus on discontinuous Galerkin formulation, which leads to block-diagonal mass matrix. We will show how to exploit the good properties of orthogonal basis functions in order to obtain efficient algorithms. The computational complexity obtained with nodal basis functions is in $O(r^6)$, where r denotes the order of approximation, whereas the computational complexity obtained with orthogonal basis functions is in $O(r^4)$. A tedious point concerns the computation and the inversion of mass matrix on each non-affine element. We will propose different solutions to this issue, and compare them. Numerical 3-D experiments will be presented for time-domain Maxwell equations in complex geometries.

Références

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