

A Posteriori Error Estimation for the Discrete Duality Finite Volume Discretization of the Stokes Equation

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Let Ω be a two dimensional polygonal domain with boundary Γ . We consider the Stokes equations [2]

$$\begin{aligned} -\Delta \hat{\mathbf{u}} + \nabla \hat{p} &= \mathbf{f} \text{ in } \Omega \\ \nabla \cdot \hat{\mathbf{u}} &= 0 \text{ in } \Omega \\ \hat{\mathbf{u}} &= 0 \text{ on } \Gamma \\ \int_{\Omega} \hat{p}(x) dx &= 0, \end{aligned}$$

where $\hat{\mathbf{u}}$ is the fluid velocity, \hat{p} the pressure and \mathbf{f} the body forces per unit mass. We derive an a posteriori error estimation for the discrete duality finite volume (DDFV) discretization of the Stokes equations on very general two-dimensional meshes, when a penalty term is added in the incompressibility equation to stabilize the variational formulation (see [3]). The main tool for the derivation of the a posteriori estimation is the (almost) equivalence of the DDFV method with a non-conforming finite element method like in [4]. The a posteriori error estimate includes 2 terms, one for the velocity and one for the pressure. The first is obtained by using a decomposition of the velocity error term like in [1]. The other one is based on the inf-sup condition. The error estimators are globally upper as well as locally lower bounds for the error of the DDFV discretization. We shall present numerical tests and especially consider the balance between the error indicator due to the discretization and the error indicator due to the penalty term.

Références

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