Simulation numérique dans l'hydrodynamique côtière

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Collaborators

ENS Cachan team :

Frédéric Dias : Professeur des Universités

Jean-Michel Ghidaglia : Professeur des Universités

Raphaël Poncet : Post-doc

Denys Dutykh : Post-doc

International collaborators :

Costas Synolakis : Professor, Univeristy of Southern California

Vassili Dougalis : Professor, University of Athens

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FIG.: Crete Island. Water waves climbing the beach near from Heraklion.

Mathematical models

Various asymptotic long wave models

Basic model :

- Full free surface potential flow equations

  — Too expansive computations

Simplification

Use shalowness parameter $\mu^2 := \frac{h^2}{\ell^2}$ to reduce the problem complexity

Zoology of long wave models

- Nonlinear shallow water equations
- Serre equations
- Boussinesq equations
- Green-Naghdi equations
- ...

Boussinesq equations

D.H. Peregrine system

- Mass conservation :

$$\partial_t H + \nabla \cdot \big(H\vec{u}\big) = 0$$

- Horizontal momentum :

$$\partial_t (H\vec{u}) + \nabla \cdot \Big(H\vec{u} \otimes \vec{u} + \frac{g}{2} H^2 \mathbb{I}\Big) + \partial_t \Big(\frac{d^3}{6} \nabla\nabla \cdot \big(\frac{H\vec{u}}{d}\big) - \frac{d^2}{2} \nabla\nabla \cdot \big(H\vec{u}\big)\Big) = gH\nabla d$$

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Definition of wave runup

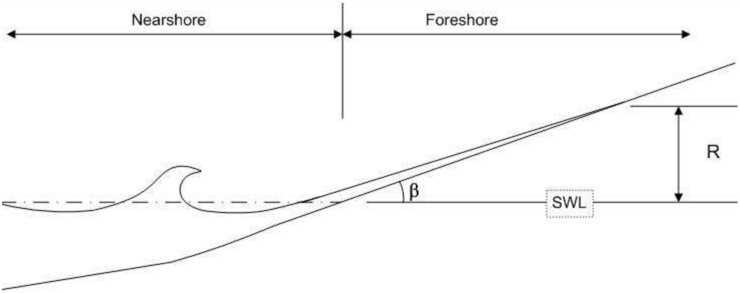

FIG.: SWL indicates Still Water Level

Definition :

Wave runup is the maximum vertical extent of wave uprush on a beach or structure above the still water level (Sorensen, 1997)

Why to model wave runup ?

Basic motivation for this study

Main interest for *tsunami people* :

- Computation of the so-called inundation maps
- Modelling wave reflection against a beach

Applications in Hydrology :

- Wave runup is an important process in bluff erosion
- Sediment transport
- Beach morphology, etc.

Engineers use empiric rules of thumb to estimate runup value

- Speaker knows about six different rules : Hunt (1959), Battjes (1974), CERC Shore Protection Manual (1984), Mase (1989), Nielsen and Hanslow (1991), Ahrens and Seelig (1996)

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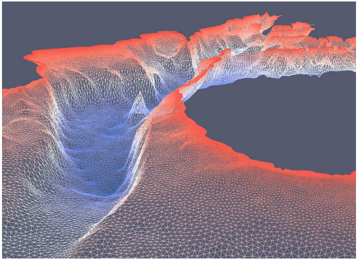
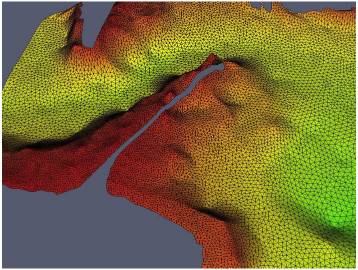
Numerical scheme used in code VOLNA

Code has been developed together with **Raphaël Poncet**

- Unstructured triangular meshes adapted upon bathymetry
- Numerical flux : VFFC, HLL, HLLC
- Second-order shock-capturing finite volume scheme
  - Least-squares gradient reconstruction method
  - Barth-Jespersen limiter
- Time stepping : SSP-RK 3(4), with $CFL = 2$





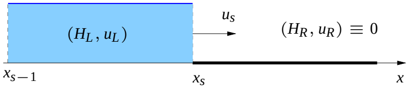
Reference :

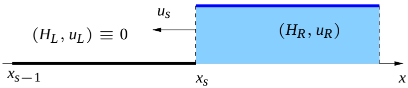
D. Dutykh. *Mathematical modelling of tsunami waves*. PhD thesis, 2007

Shoreline Riemann problem

Algorithm used in our code VOLNA is based on this analytical result

- Method of charecteristics
- Riemann invariants





Runup

$$\frac{dx_s}{dt} = u_s, \quad u_s := u_L + 2c_L$$

Rundown

$$\frac{dx_s}{dt} = u_s, \quad u_s := u_R - 2c_R$$

Gravity wave speed : $c_{L,R} := \sqrt{gH_{L,R}}$

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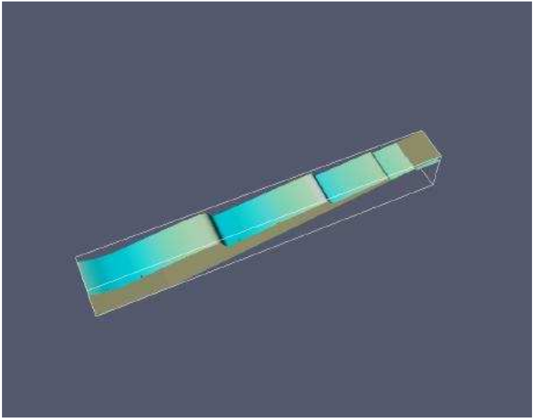
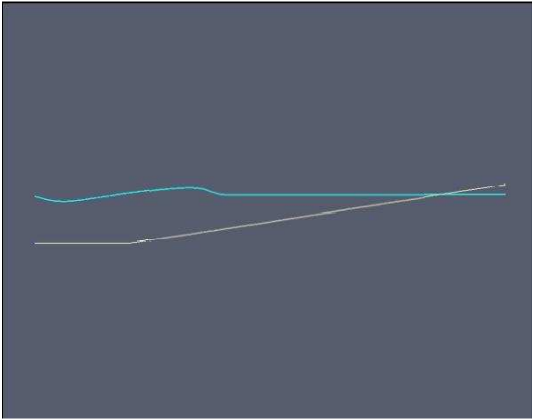
Periodic wave runup on a plane beach - I

3D view (by courtesy of R. Poncet)



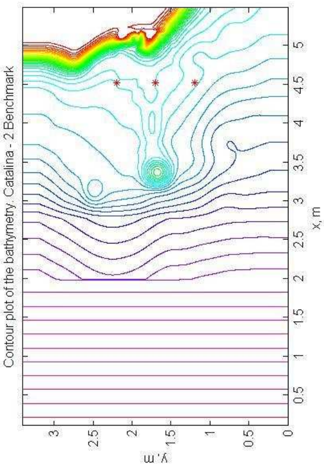
Periodic wave runup on a plane beach - II

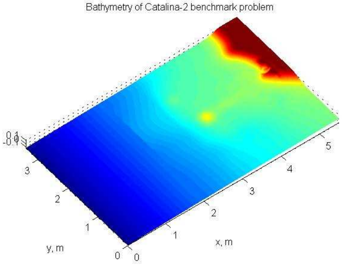
Horizontal cut view (by courtesy of R. Poncet)



Catalina # 2 benchmark problem

Tsunami run-up on a complex island





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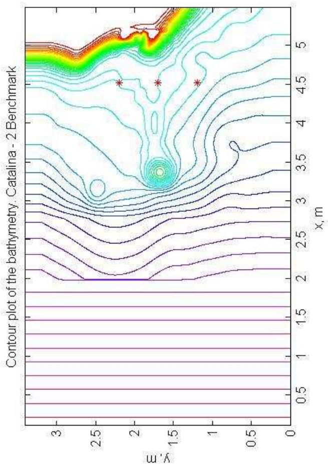
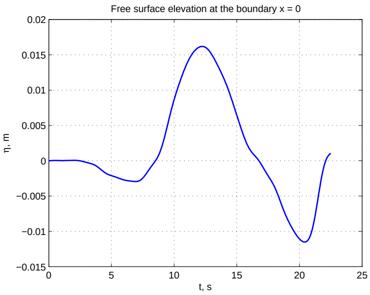
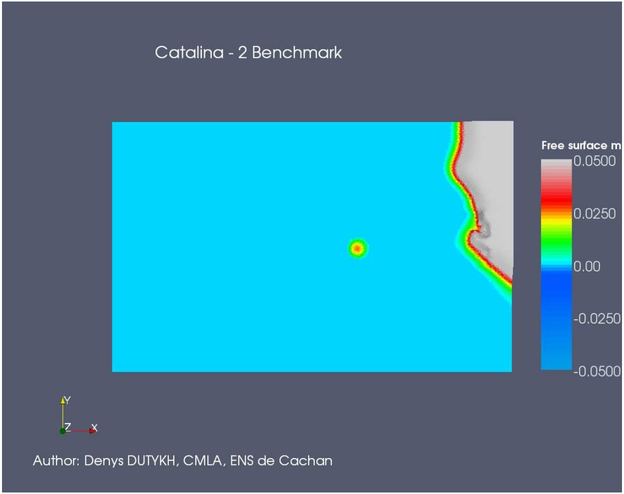




Catalina # 2 simulation results

Free surface elevation



Comparison with experiments

Tide gauge data at different locations

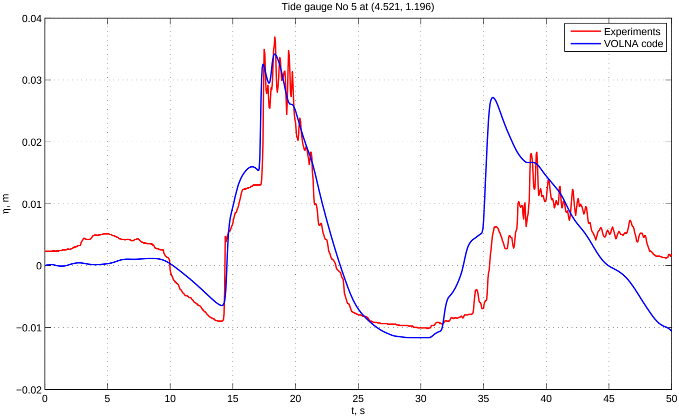


FIG.: Tide gauge #5

Comparison with experiments

Tide gauge data at different locations

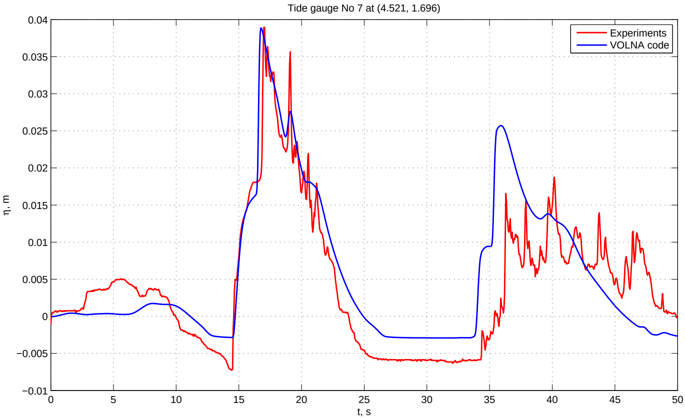


FIG.: Tide gauge #7

Comparison with experiments

Tide gauge data at different locations

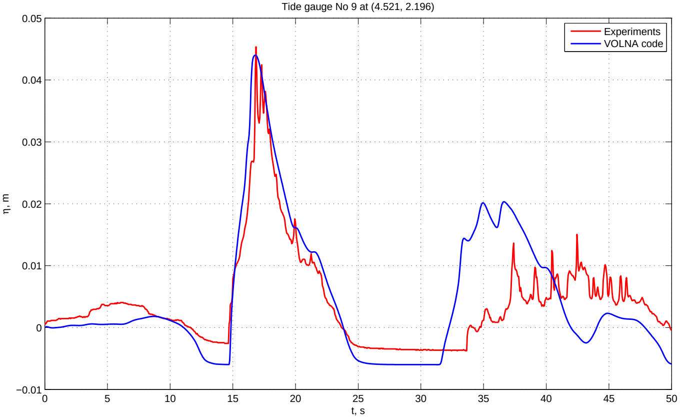


FIG.: Tide gauge #9

Comparison with CLAWPACK

Results by R. LeVeque and D. George

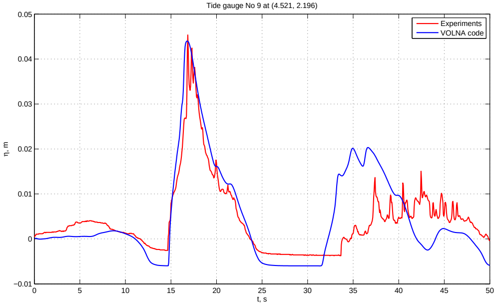


FIG.: VOLNA code

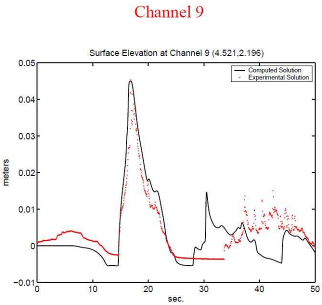


FIG.: CLAWPACK

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Perspectives for future work

- Inclusion of dispersive terms (Boussinesq solver) . . .
- Domain decomposition + MPI (basically R. Poncet's task)

Thank you for your attention !

http://www.cmla.ens-cachan.fr/~dutykh