

A discontinuous Galerkin scheme for a stratigraphic model

Abdelaziz Taakili, LMA, Pau

Roland Becker, LMA, Pau

Guy Vallet, LMA, Pau

Mots-clés : Pseudo-parabolic problem, degenerated problem, discontinuous Galerkin methods.

In this talk, we study a mathematical problem arising from the modelling of maximal erosion rate in geological stratigraphic phenomena. The equation of this problems is nonlinear; the diffusion coefficient depends on the time-derivative of the unknown u and degenerates in order to take implicitly into account a global constraint on the time-derivative of u . This model has been initially developed by the Institut Français du pétrole and it takes into account sedimentation, transport, accumulation and erosion phenomena. Concerning physical and numerical description of these models, see in R. Eymard and al. [6], [5]. The original mathematical aspect of this model is the imposition of a constraint on the time-derivative of u . This leads us to consider a class of conservation laws of degenerate pseudo-parabolic type that occurs in particular in the theory of elastic fluids in elasto-plastic porous media (see G. I. Barenblatt [3])

$$\partial_t u - \text{Div}[\lambda K(x) \nabla(u + \tau \partial_t u)] = 0, \quad \lambda \in H(\partial_t u + E), \quad (1)$$

where, H denotes the maximal monotone graph of the function of Heaviside, u denotes the sediment thickness and E denotes the admissible erosion rate. It contains implicitly the constraint

$$\partial_t u + E \geq 0. \quad (2)$$

Existence and uniqueness results of the solution to the above differential inclusion are still open problems. A modified model where H is replaced by a Lipschitz continuous function a is analysed by S. N. Antontsev, G. Gagneux, R. Luce and G. Vallet [1]. An existence result of a solution to this nonlinear degenerate pseudoparabolic equation by the way of a compactness argument is considered when $K = 1$. In particular, local hyperbolic behavior of solutions is proved. Our purpose in this talk is, on the one hand, to extend to more general assumptions on the data, the results of the above cited paper, in particular with a source term f . On the other hand, we introduce the discontinuous Galerkin method for the model problem. The considered DG methods is a variant of the Symetric Interior penalty Galerkin method (SIPG). The difference appears in the consistency terms where the arithmetic average used in SIPG method is replaced by the weighted dependency on a . With this particular choice, we prove that the piecewise constant discontinuous Galerkin scheme satisfies implicitly the constraint. Some properties of the scheme are established. The theoretical results are completed by some numerical results.

Références

- [1] S. N. Antontsev, G. Gagneux, R. Luce, and G. Vallet. On a pseudoparabolic problem with constraint. *Differential and Integral Equations*, 19(12):1391–1412, 2006.
- [2] D.N Arnold. An interior penalty finite element methods with discontinuous elements. *SIAM J. Numer. Anal.*, 19(4):742–760, 1982.
- [3] G. I. Barenblatt. Similarity, self-similarity, and intermediate asymptotics. *New York, London: Consultants Bureau. XVII*, 1982.
- [4] R. Becker, P. Hansbo, and R. Stenberg. A finite element method for domain decomposition with non-matching grids. *M2AN*, 37(2):209–225, 2003.
- [5] R. Eymard and T. Gallouët. Analytical and numerical study of a model of erosion and sedimentation. *SIAM J. Numer. Anal.*, 43(6):2344–2370, 2006.
- [6] R. Eymard, T. Gallouët, D. Granjeon, R. Masson, and Q.H. Tran. Multilithology model under maximum erosion rate constraint. *Internat. J. Numer. Methods Engrg.*, 60(2):527–548, 2004.