The Evans function for the nerve influx

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The system of equations with models the propagation of the nerve influx is a system of ordinary differential equations modelizing the exchange of ions through the membrane of the axon coupled with the partial differential equation satisfied by the mean of the electric axoplasm. This partial differential equation is an approximation induced by the fact that the diameter of the axon is much smaller than its length.
As the exchange of ions depends strongly of the difference of potential across the membrane, the system of ordinary differential equations is nonlinear.
The resulting system is called the Hodgkin-Huxley system [5].
We seek a progressive solution of this system, and we define and evaluate the Evans function (generalization of the Lopatinskii determinant associated with the linearized system) for the study of the stability of this progressive solution, [1], [2], [3], [4].
We explain the link with the Evans function for hydrodynamic instabilities [6], [7] and write a new Evans function for a modified Hodgkin-Huxley system which admits two stationary solutions.

Références

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