Stanislav Smirnov

Citation: “For the proof of conformal invariance of percolation and the planar Ising model in statistical physics.”

Stanislav Smirnov has put a firm mathematical foundation under a burgeoning area of mathematical physics. He gave elegant proofs of two long-standing, fundamental conjectures in statistical physics, finding surprising symmetries in mathematical models of physical phenomena.

Though Smirnov’s work is highly theoretical, it relates to some surprisingly practical questions. For instance, when can water flow through soil and when is it blocked? For it to flow, small scale pores in the soil must link up to provide a continuous channel from one place to another. This is a classic question in statistical physics, because the large-scale behavior of this system (whether the water can flow through a continuous channel of pores) is determined by its small-scale, probabilistic behavior (the chance that at any given spot in the soil, there will be a pore).

It’s also a natural question to model mathematically. Imagine each spot in the soil as lying on a grid or lattice, and color the spot blue if water can flow and yellow if it can’t. Determine the color of each spot by the toss of a coin (heads for yellow, tails for blue), using a coin that might be weighted rather than fair. If a path of blue spots crosses from one side of a rectangle to the other, the water can pass from one side to the other.

Such “percolation models” behave in a remarkable way. For extreme values, the behavior is as you might expect: If the coin is heavily weighted against blue, the water almost certainly won’t flow, and if it’s heavily weighted toward blue, the water almost certainly will. But the probability of flow doesn’t change evenly as the percentage of blue spots increases. Instead, the water is almost certainly going to be blocked until the percentage of blue spots reaches some threshold value, and once it does, the probability that the water will flow starts surging upward. This threshold is called the “critical point.” Abrupt change of behavior like this is a bit like what happens to water as it heats: suddenly, at a critical temperature, the water boils. For that reason, this phenomenon is commonly called a phase transition.

But of course, real soil doesn’t come with neat, evenly spaced horizontal or vertical pores. So to apply this model to the real world, a couple of troublesome questions arise. First, how fine should the lattice grid be? Physicists are most interested in understanding processes at the molecular scale, in which case the grid should be very small indeed. Mathematicians then ask about the relationship between models with ever-smaller grids. Their hope is that as the grids get finer, the models will get closer and closer to one single model that effectively has an infinitely fine grid, called a “scaling limit.”

To see why it’s not obvious that the scaling limit will exist, imagine choosing a particular percentage of blue spots for a lattice and calculating the probability that the pores will line up to form a crossing. Then make the grid size smaller and calculate it again. As the grids get finer, the crossing probabilities may get closer and closer to some number, the way the numbers 1.9, 1.99, 1.999, 1.9999… get closer and closer to 2. In that case, this number will be the crossing probability for the scaling limit. But it's imaginable that the crossing probabilities will jump around and never converge toward a limit, like the sequence of numbers 2, 4, 2, 4, 2, 4… In that case, should the crossing probability for the scaling limit be 2 or 4? There's no good answer, so we have to say that the scaling limit doesn't exist.

Another potentially problematic question is what shape lattice to use. Even if we restrict ourselves to two dimensions, there are many choices: square lattices, triangular lattices, rhombic lattices… Ideally, the model would be "universal," so that the choice of lattice shape doesn't matter, but that's not obviously true.

Physicists are pretty sure that neither of these potential problems is so bad. Using physical intuition, they've argued convincingly that the model will indeed approach a well-defined scaling limit as the grid gets finer. Furthermore, though the choice of lattice shape does affect the critical point, physicists have persuaded themselves that it won't affect many of the other properties they're interested in.

Physicists have figured out even more about two-dimensional lattices, including finding evidence for a surprising and beautiful symmetry. Imagine taking a lattice of any shape and stretching it or squinching it but leaving the angles all the same. The Mercator projection of the globe is an example of this: Greenland is huge since distances are changed, but latitude and longitude lines nevertheless stay at right angles. Physicists have convinced themselves that if you transform two-dimensional percolation models in this way, it won't change their scaling limits (as long as you're near the critical points). Or, to use the technical term, they're persuaded that scaling limits are "conformally invariant."

In 1992, John Cardy, a physicist at the University of Oxford, used this insight to achieve one of percolation theory's big goals: a precise formula that calculates the crossing probabilities of the scaling limits of two-dimensional lattices near the critical point. The only problem was that although his physical arguments were persuasive, neither he nor anyone else could turn that physical intuition into a mathematical proof.

In 2001, Smirnov put all this physical theory on a firm mathematical foundation. He proved that scaling limits are conformally invariant, though only for the triangular lattice (the shape that pennies, for example, fall into naturally when laid flat on a table and packed tightly together). In the process, he also proved the correctness of Cardy's formula for triangular lattices. His proof used an approach independent of ones used earlier by physicists that provided fundamental new insights. It also provided a critical missing step in the theory of Schramm–Loewner Evolution, an important, recently developed method in statistical physics.

In another major achievement, Smirnov used similar methods to understand the Ising model, which describes phenomena including magnetism, gas movement, image processing, and ecology. Just as with percolation, the large-scale behaviors of these phenomena are determined by their probabilistic, small-scale behavior. Consider, for example, magnetism: The atoms in a piece of iron behave like miniature magnets, with the electrons moving around the nucleus creating a miniature magnetic field. The atoms try to pull their neighbors into the same alignment as their own. When enough atoms have their north poles pointing the same direction, the iron as a whole becomes magnetic. Mathematicians model this by visualizing the atoms as lying on the nodes of a lattice, with statistical rules that determine whether they're aligned with their north poles pointing up or down.

Like the percolation model, the Ising model undergoes a phase transition: As you heat the iron, the atoms vibrate more quickly, and if you heat it above a certain point, the vibrations are so strongly that neighboring atoms suddenly no longer hold one another in alignment and the piece as a whole begins to lose its magnetism.

The same questions that mathematicians and physicists worry about in percolation also apply to the Ising model. The grid should be extremely small, since it's operating on the atomic level. So as the grid mesh gets finer and finer, does the model converge toward some infinitely fine version, a scaling limit? Furthermore, how does the lattice shape affect the critical point and other properties? And what happens if you stretch or squish the lattice without changing the angles – does the scaling limit change?

For this model, too, Smirnov was able to show that the models do indeed converge toward a scaling limit as the grid mesh gets finer and that they are unaffected by stretches and squishes, i.e., that they are conformally invariant. Later, with Dmitry Chelkak, he established universality, extending the results to a wide range of different lattices. He has also done significant work in analysis and dynamical systems. His work will continue to enrich both mathematics and physics in the future.

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