Modélisation du parallélisme et de la synchronisation

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Vision

- **Study the temporal behavior of interacting and parallel components systems**
- **Objective : model for performance analysis and prediction : Stochastic Automata Networks**
- **Target for a specification language**

- **Challenges :**
- **Handle the information size explosion (State based approach and Markov theory)**
- **With composability**

Performance of computer systems

- Queuing networks [Little, Jackson, Gordon, Newell, Chandy et all, Kelly, Kleinrock, Gelenbe,Grassmann, Kobayashi, Lavenberg, Reiser, Sevcik, Latouche, Marie, Stoyan, Muntz, Mitrani, Baccelli, Nain, Trivedi, Pujolle, Jain, Lazowska, King, Zahorjan…]
- Markov Chains [Stewart, Courtois, Golub, Meyer, Neuts…]
- State based approach :
  - Petri Nets [Chiola, Ciardo, Donatelli, Buchhloz, Marsan, Miner, Ciardo, Molloy, Vernon, Trivedi, Sanders, Meyer, Buchholz, Kemper …]
  - Performance Evaluation Process Algebra [Hillston, Hermanns]
  - Structures Domains [Maler]
  - Stochastic Automata Networks (SANs) […

Applications

- **Parallel Architecture [Touzene]**
- **Distributed algorithms and systems [Tripathi, Bolot]**
- **Networks [Fourneau, Veque, Kloul, Pekergin, Valois]**
- **Road traffic [Junblut]**
- **Flexible manufacturing systems [Benoit]**
- **Reliability [Brenner, Fernandes]**
- **Biology and chemistry [Wolf, Nguyen, Roy Mathias, Smith]**

- **Partners : IBM, Bull, France Telecom**
- **A tool PEPS**

Outline

- **Modelling approach**
- **Generalized tensor products**
- **Linear algebra**
- **Extensions : symetry, distribution**
- **Product forms**
- **Current work**

- **Not here … Data structures and reachable state space, Numerical analysis, Bounds, Simulation, graph analysis, hierarchy, decomposability, reductions, approximations, etc.**

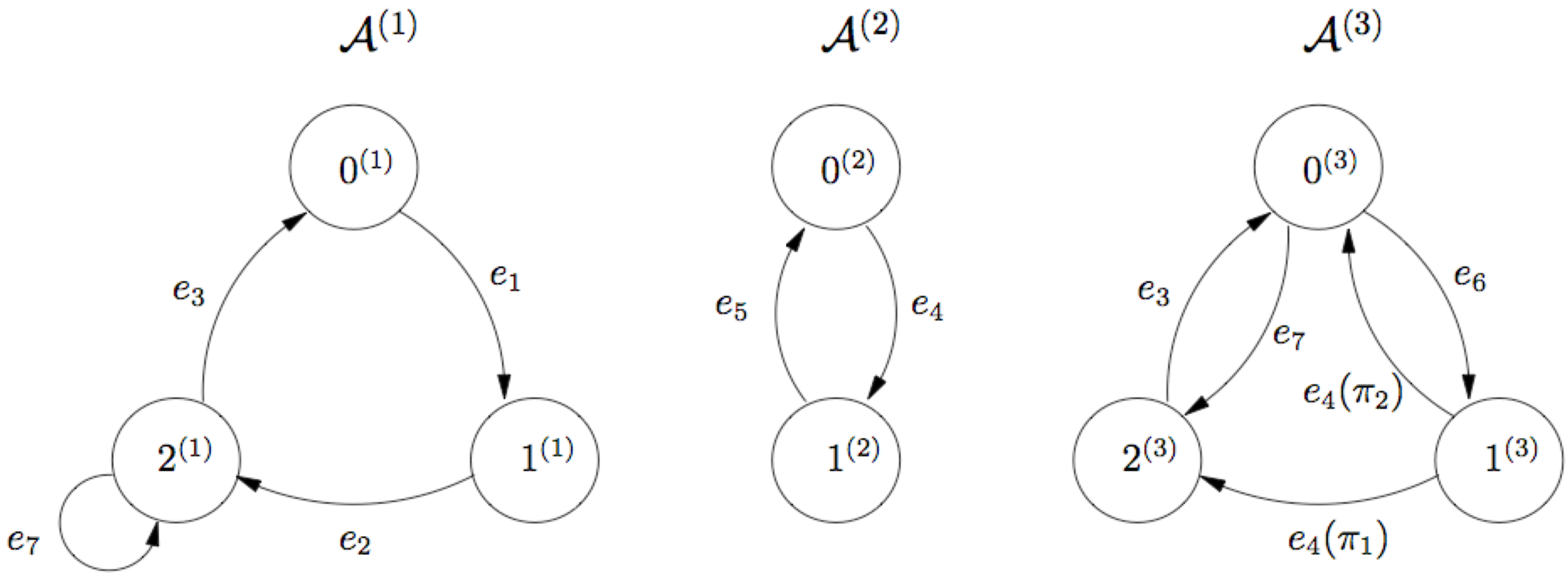
SAN Modeling : Principles

- **Continuous time Markov theory**
- **Describe large Markov chains**

- **Each component is represented by a stochastic automaton (state, transition, events with rates of exponential distribution, routing probabilities)**
- **Local events change the state of only one automaton**
- **Synchronized events change the state of more than one automaton**

- **Functions for rates describe complex interactions among components.**

SAN Modeling :Example

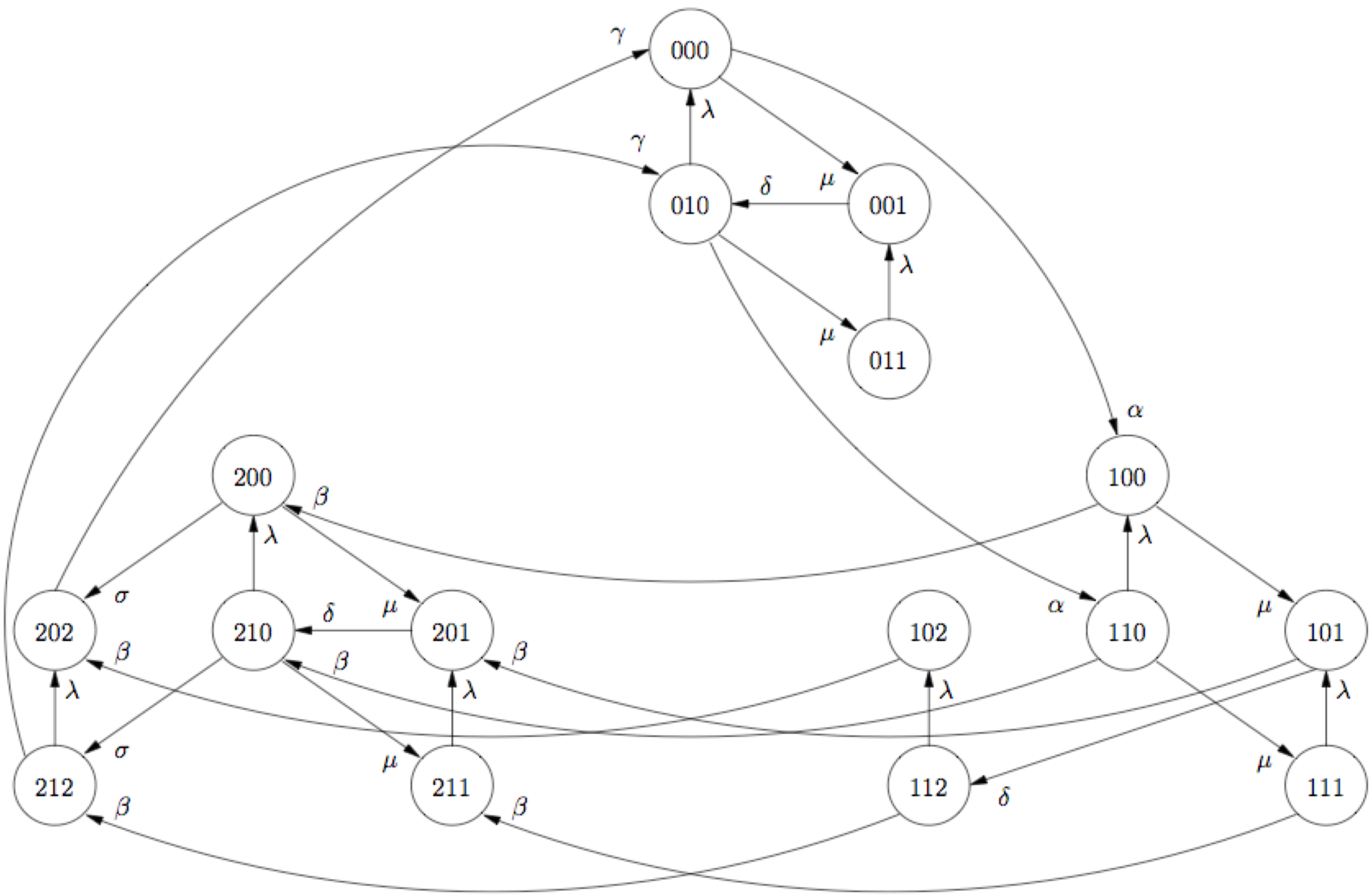


$f_1 = (st\mathcal{A}^{(3)} == 0) * \alpha$ $\quad \pi_1 = (st\mathcal{A}^{(1)} == 1)$ $\quad \pi_2 = 1 - \pi_1$

Type	Event	Rate	Type	Event	Rate	Type	Event	Rate
loc	e_1	f_1	syn	e_4	δ	loc	e_6	μ
loc	e_2	β	loc	e_5	λ	syn	e_7	σ
syn	e_3	γ						

SAN Modeling : the Markov Chain

global state, combination of the local states of each automata, on the RSS



A
S

SAN Definition

A SAN model is defined by $\mathcal{M} = (N, \mathcal{A}, \mathcal{E}, f)$

- N the number of automata
- $\mathcal{E}$ the set of events $e = (loc \mid syn, \tau_e) \in \mathcal{E}$
- $\mathcal{A} = \{\mathcal{A}^{(1)}, \mathcal{A}^{(2)}, ..., \mathcal{A}^{(N)}\}$ the set of automata
  $\forall i, \mathcal{A}^{(i)} = (\mathcal{S}^{(i)}, \mathcal{Q}^{(i)})$, where $\mathcal{Q}^{(i)} : \mathcal{S}^{(i)} \times \mathcal{S}^{(i)} \rightarrow \mathcal{T}^*$
  and $\mathcal{T}$ the set of tuples (e, π_e), $e \in \mathcal{E}$
  $\hat{\mathcal{S}} = \mathcal{S}^{(1)} \times \mathcal{S}^{(2)} \times ... \times \mathcal{S}^{(N)}$ the product state space
- $\mathcal{F}$ the set of functions from $\hat{\mathcal{S}}$ to $\mathcal{R}^+$
  $\mathcal{S} = \{\tilde{x} \in \hat{\mathcal{S}} | f(\tilde{x}) = 1\}$ the reachable state space
  the τ_e and π_e are in $\mathcal{F}$, and $\pi_e \in [0..1]$

Markovian Descriptor

- **SAN: modular description and manipulation. Generator of the Markov chain = descriptor Q**

$$Q = \bigoplus_{g\,i=1}^{N} Q_l^{(i)} + \sum_{e \in \mathcal{ES}} \bigotimes_{g\,i=1}^{N} Q_{e^+}^{(i)} + \bigotimes_{g\,i=1}^{N} Q_{e^-}^{(i)}$$

  - **Tensor product: synchronising part - E sync events.**

- **Generalized tensor algebra (functional matrices) [Plateau, Fernandes, Stewart, 98 ...].**
- **Memory efficient - N components of size *ni*:**

$$\prod_{i=1}^{N} n_i^2 \quad \longrightarrow \quad \sum_{i=1}^{N} n_i^2$$

Tensor approach : the product

Product A ≅ B =

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cong \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$= \left(\begin{array}{ccc|ccc} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & a_{12}b_{11} & a_{12}b_{12} & a_{12}b_{13} \\ a_{11}b_{21} & a_{11}b_{22} & a_{11}b_{23} & a_{12}b_{21} & a_{12}b_{22} & a_{12}b_{23} \\ a_{11}b_{31} & a_{11}b_{32} & a_{11}b_{33} & a_{12}b_{31} & a_{12}b_{32} & a_{12}b_{33} \\ \hline a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} & a_{22}b_{11} & a_{22}b_{12} & a_{22}b_{13} \\ a_{21}b_{21} & a_{21}b_{22} & a_{21}b_{23} & a_{22}b_{21} & a_{22}b_{22} & a_{22}b_{23} \\ a_{21}b_{31} & a_{21}b_{32} & a_{21}b_{33} & a_{22}b_{31} & a_{22}b_{32} & a_{22}b_{33} \end{array}\right)$$

Generalized tensor approach

$$A \cong_g B = \begin{pmatrix} a_{11}(k) & a_{12}(k) \\ a_{21}(k) & a_{22}(k) \end{pmatrix} \cong_g \begin{pmatrix} b_{11}(h) & b_{12}(h) & b_{13}(h) \\ b_{21}(h) & b_{22}(h) & b_{23}(h) \\ b_{31}(h) & b_{32}(h) & b_{33}(h) \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc}
a_{11}(1)\,b_{11}(1) & a_{11}(1)\,b_{12}(1) & a_{11}(1)\,b_{13}(1) & a_{12}(1)\,b_{11}(1) & a_{12}(1)\,b_{12}(1) & a_{12}(1)\,b_{13}(1) \\
a_{11}(2)\,b_{21}(1) & a_{11}(2)\,b_{22}(1) & a_{11}(2)\,b_{23}(1) & a_{12}(2)\,b_{21}(1) & a_{12}(2)\,b_{22}(1) & a_{12}(2)\,b_{23}(1) \\
a_{11}(3)\,b_{31}(1) & a_{11}(3)\,b_{32}(1) & a_{11}(3)\,b_{33}(1) & a_{12}(3)\,b_{31}(1) & a_{12}(3)\,b_{32}(1) & a_{12}(3)\,b_{33}(1) \\
\hline
a_{21}(1)\,b_{11}(2) & a_{21}(1)\,b_{12}(2) & a_{21}(1)\,b_{13}(2) & a_{22}(1)\,b_{11}(2) & a_{22}(1)\,b_{12}(2) & a_{22}(1)\,b_{13}(2) \\
a_{21}(2)\,b_{21}(2) & a_{21}(2)\,b_{22}(2) & a_{21}(2)\,b_{23}(2) & a_{22}(2)\,b_{21}(2) & a_{22}(2)\,b_{22}(2) & a_{22}(2)\,b_{23}(2) \\
a_{21}(3)\,b_{31}(2) & a_{21}(3)\,b_{32}(2) & a_{21}(3)\,b_{33}(2) & a_{22}(3)\,b_{31}(2) & a_{22}(3)\,b_{32}(2) & a_{22}(3)\,b_{33}(2)
\end{array}\right)$$

Properties of generalized tensor products

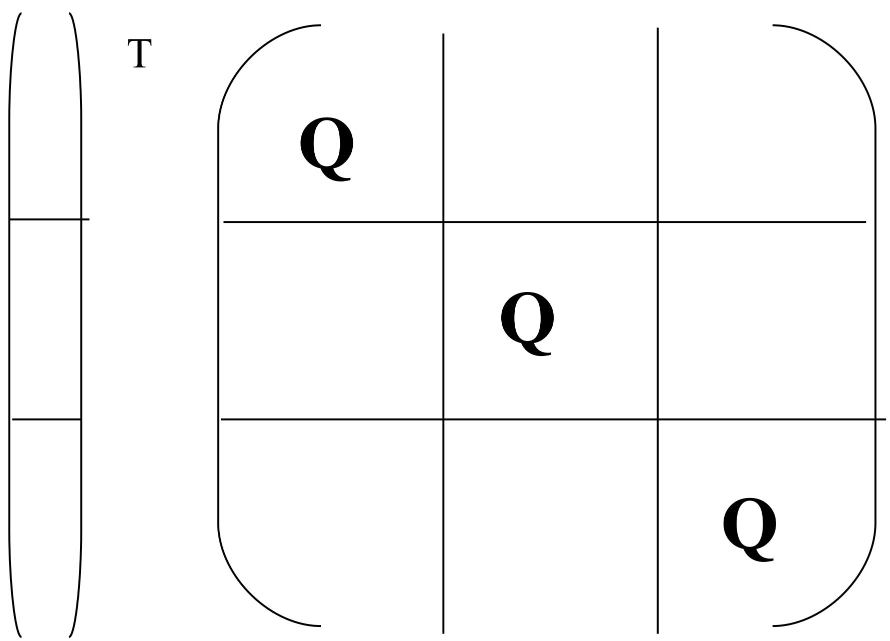
- It keeps almost all algebraic properties of the ordinary tensor product as :
- Associativity
- $(A+C) \cong B = (A \cong B) + (C \cong B)$
- $(A \cong B)^T = A^T \cong B^T$ & $(A \cong B)^{-1} = A^{-1} \cong B^{-1}$
- $(A \cong B) = (A \cong Idn_B).(Idn_A \cong B)$ with an ordering restriction
- $A \cong B = Perm\ (B \cong A)\ Perm^T$, pseudo-commutativity
- A GTP is decomposable in a sum of OTP

But not

- $(A \cong Idn_B).(Idn_A \cong B)=(Idn_A \cong B).(A \cong Idn_B)$, commutativy of normal factors
- $(A . B) \cong (C . D) = (A \cong C) . (B \cong D)$
- Eigenvalues properties

$v \cong {}_{g\ i=1\ldots N}\ Q_i$: normal factor and shuffle

- $v\ (Id_3 \cong\ Q) =$



A Normal Factor (NF) : Complexity size(Id).size(Q)^2 -

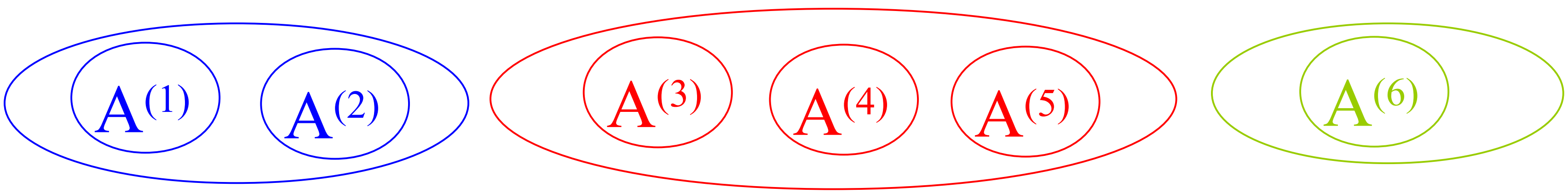
Shuffle algorithm v ≅ $_{g\ i=1\ldots N}$ Q_i

Optimizations

- Compute on the reachable (not product) state space
- Matrix sparsity
- Optimize the shuffle step
- Optimize function evaluation time which might be dominant by reordering the blocks so that identical blocks evaluations are processed in sequence
- adapted numerical method (block, hierarchique, preconditionning, …)

State space reduction : aggregation

- **using replications of components and exact lumping [Kemeny, Snell], [Siegle],[Buchhloz], equivalence [Buchhloz],**



- **[Benoit]: Conditions on the functions : given ≅ ≅ which invariant the partition:**f(x) = f(≅ (x)).

$$s(x) = s(x^{(1)},\dots,x^{(N)}) = (x^{s(1)},\dots,x^{s(N)})$$

=> Descriptor for the aggregated model

Releasing the exponential assumption

- Exponential distributions, general distributions, Phase-Type distribution

- Queueing Networks: [M.Neuts, P. Buchholz and others].
- Petri Nets: The regeneration points [Ajmone Marsan and others]
- Conditions on the model PH transitions are replaced by a sub-net [Molloy and others].
- PH distributions is taken into account during the generation of the reachable state [Cumani].
- Structural conditions [Haddad, Moreaux] to obtain a descriptor

Product forms : sufficient conditions

- find $\pi = (\prod_{i=1\ldots N} \pi_i)$ or $\pi = (\prod_{i=1\ldots N} \pi_i)/C$

First context [Boucherie, Robertazzi, Stewart et al.]

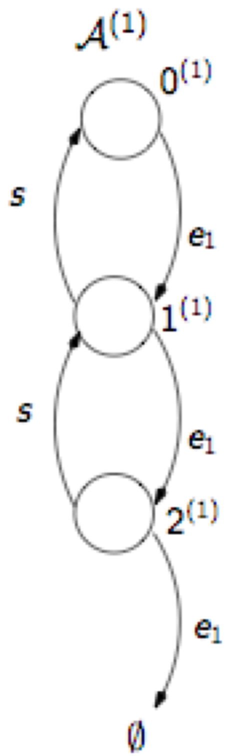
- No synchronizing event
- local balance :the functions express a truncation of the state space or the functions modify uniformly the rates of one automata, or both
- More general : kernels of submatrices with common eigenvectors

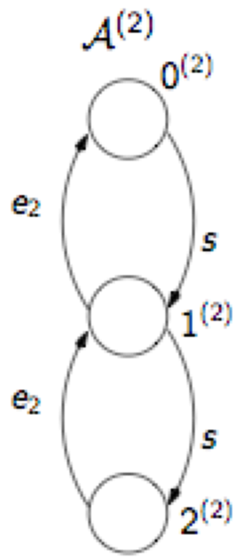
Second context [Fourneau et al.] :

- Limited synchronization schemas :
- 2 partners RV
- Domino synchronisation
- Fixed point system

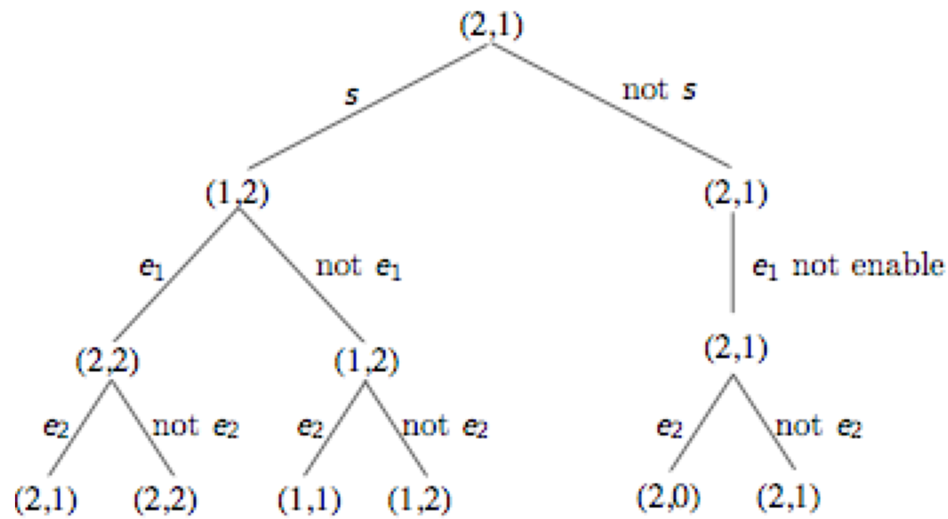
Current work : Discrete time

- **Semantics : state, transition, probability of occurrence, simultaneous events and priority, [Benoit]**





Type	Event	Prob.
loc	e_1	λ
loc	e_2	μ
syn	s	α



Discrete time : Descriptor structure

$$P = P^{(1)}_{e_1,e_2} \otimes_g P^{(2)}_{e_1,e_2} + P^{(1)}_{s} \otimes_g P^{(2)}_{s} + P^{(1)}_{\bar{s}} \otimes_g P^{(2)}_{\bar{s}}$$

$$P^{(1)}_{e_1,e_2} = \begin{pmatrix} (1-\lambda)f_1 & \lambda f_1 & 0 \\ 0 & (1-\lambda)f_1 & \lambda f_1 \\ 0 & 0 & f_1 \end{pmatrix} \quad P^{(2)}_{e_1,e_2} = \begin{pmatrix} f_2 & 0 & 0 \\ \mu f_2 & (1-\mu)f_2 & 0 \\ 0 & \mu f_2 & (1-\mu)f_2 \end{pmatrix}$$

$$P^{(1)}_{s} = \begin{pmatrix} 0 & 0 & 0 \\ \alpha(1-\lambda) & \alpha\lambda & 0 \\ 0 & \alpha(1-\lambda) & \alpha\lambda \end{pmatrix} \quad P^{(2)}_{s} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \mu & (1-\mu) \\ 0 & 0 & 0 \end{pmatrix}$$

$$P^{(1)}_{\bar{s}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (1-\alpha)(1-\lambda) & (1-\alpha)\lambda \\ 0 & 0 & (1-\alpha) \end{pmatrix} \quad P^{(2)}_{\bar{s}} = \begin{pmatrix} 1 & 0 & 0 \\ \mu & (1-\mu) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

With $f_1 = f_2 = 1_{\{0^{(1)}0^{(2)},\ 0^{(1)}1^{(2)},\ 0^{(1)}2^{(2)},\ 1^{(1)}2^{(2)},\ 2^{(1)}2^{(2)}\}}$

[Brenner] : open problem, formalize a tensor operator and its properties

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Thank you for your attention and if you have questions….