Model order reduction methods for parameter-dependent equations – Applications in Uncertainty Quantification.

PhD Thesis

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Model order reduction has become an inescapable tool for the solution of high dimensional parameter-dependent equations arising in uncertainty quantification, optimization or inverse problems. In this thesis we focus on low rank approximation methods, in particular on reduced basis methods and on tensor approximation methods.

The approximation obtained by Galerkin projections may be inaccurate when the operator is ill-conditioned. For projection based methods, we propose preconditioners built by interpolation of the operator inverse. We rely on randomized linear algebra for the efficient computation of these preconditioners. Adaptive interpolation strategies are proposed in order to improve either the error estimates or the projection onto reduced spaces. For tensor approximation methods, we propose a minimal residual formulation with ideal residual norms. The proposed algorithm, which can be interpreted as a gradient algorithm with an implicit preconditioner, allows obtaining a quasi-optimal approximation of the solution.

Finally, we address the problem of the approximation of vector-valued or functional-valued quantities of interest. For this purpose we generalize the 'primal-dual' approaches to the non-scalar case, and we propose new methods for the projection onto reduced spaces. In the context of tensor approximation we consider a norm which depends on the error on the quantity of interest. This allows obtaining approximations of the solution that take into account the objective of the numerical simulation.

Keywords : Model order reduction – Uncertainty quantification – Parameter dependent equations – Reduced Basis – Low rank tensor approximation – Preconditioner – Quantity of interest